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On the Flicker Noise of Ferrite Circulators for Ultra-Stable Oscillators

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Abstract

The flicker noise of the ferrite circulator is a critical element in ultra-stable microwave oscillators, in which the signal reflected from the input of the reference cavity is exploited to stabilize the frequency. This article explains why the circulator noise must be measured in isolation mode, proposes a measurement scheme, and provides experimental results. The observed flicker spans from -162 to -170 dB[rad^2]/Hz at 1 Hz off the 9.2 GHz carrier, and at +19 dBm of input power; in the same conditions, the instrument limit is below -180 dB[rad^2]/Hz. In addition, the scheme can be used as the phase detector of a corrected oscillator, and in the field of solid-state physics as an instrument for the measurement of random fluctuation in magnetic materials.

1 Statement of the Problem

The analysis of ultra-stable microwave oscillators reveals that the present configurations rely upon two basic schemes, proposed by Pound [1] and by Galani [2]. In the Pound scheme (Fig. 1 A), an oscillator is frequency locked to a reference resonator by measuring the frequency error, which is obtained from the signal reflected by the resonator. In the Galani scheme (Fig. 1 B), a reference resonator is used in transmission mode to loop the signal back to the input of the sustaining amplifier. Besides, the signal reflected by the resonator provides the error signal used to correct for the amplifier noise. A mixed configuration is sometimes used, which consists of a Galani oscillator in which the detection system is replaced with that of the Pound scheme. Looking at Fig. 1, one may object that the noise degeneration loop stabilizes a specific point, for the placement of the output tap is a critical issue. In practice this is not relevant because the feedback loop corrects for the frequency noise of the oscillator, regardless of where the output is picked up.

The main idea of the stabilized oscillator is that an error signal, obtained from the microwave signal reflected by the reference resonator, is used to correct the oscillator frequency through an appropriate noise degeneration circuit that exploits the stability of the resonator. To do so, reflection is always preferred to transmission, and a ferrite circulator is usually em-

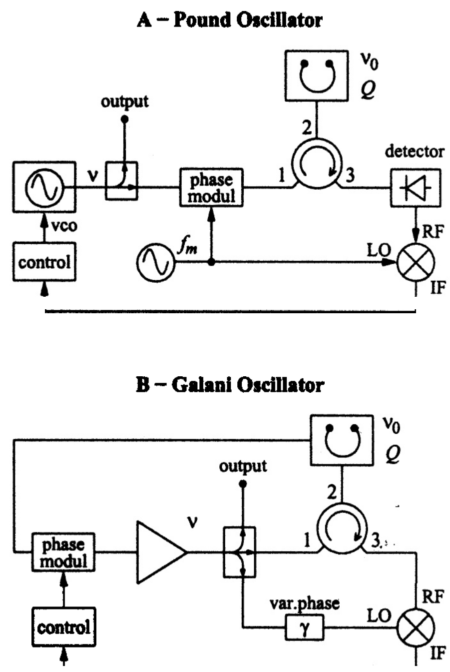

Figure 1: Ultra-stable microwave oscillators.

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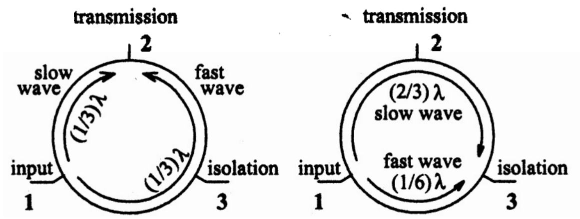

Figure 2: Circulation effect.

ployed to extract the reflected signal. The reasons for this choice are simplicity, and the low loss and the low white noise of the circulator. Yet, the flicker noise of the circulator is a good candidate to limit the oscillator stability through the phase-to-frequency conversion mechanism known as the Leeson model [3].

The circulator consists of a ferrite cylinder with three ports at 120°, biased by an axial dc field that induces the Larmor precession. The biased ferrite is gyrotropic, which means that the magnetic permeability μ seen by the waves propagating circularly in the cylinder splits into μ^- and μ^+, depending on the wave direction with respect to the precession. Wave velocity is affected accordingly, in the same way as mechanical stiffness affects the speed of sound. Circulation results from the interference between the precession-wise and counter-precession-wise modes, which is additive at the tranmission port and destructive at the isolation port, as shown in Fig. 2. Yet, reality is more complex than in that figure because circulation results from the field in the whole volume of the ferrite. A detailed analysis of the circulators can be found in the early references [4, 5], while modern devices are described in [6, 7]. Finally, if one port is internally terminated, the device is an isolator.

A bibliographical research shows that little attention has been given to noise, and that only the theory of thermal noise has been developed, based on thermal equilibrium and on the properties of the scattering matrix [8, 9]. No theory of the flicker noise has been found. Reference [10] suggests that flickering is connected to the reflection coefficient Γ. Two articles [11, 12] report experimental data on a circulator used as an isolator. Yet, the suitability of these results to the noise-corrected oscillator is arguable. We are mainly interested to low-frequency noise components, below the cutoff frequency $f_L = \frac{\nu_0}{2Q}$ of the resonator, which are the most relevant ones to the oscillator stability. Looking at Fig. 1, noise present at port 3 goes to the input of the phase detector, hence it contributes to the frequency instability. Conversely, the noise present at port 2 has the same effect of a noise source in series to the VCO output (Pound), or in series to the sustaining amplifier (Galani), for it is removed by the control loop in its normal operation. Therefore, having in mind the application in ultra-stable oscillators, the circulator noise must be measured at the isolation port, instead of the transmission port.

2 Measurement Method

The circulator is based on interference, thus an interferometric measurement sounds obvious. Figure 3 shows the measurement system, which is a multiple carrier suppression interferometer with vector detection of noise. The dashed frame encloses the test circuit that simulates the circulator insertion in an oscillator. The 16 dB attenuator and the short circuit replace a cavity whose reflection coefficient is $|S_{11}| = -32$ dB, which is typical of our room-temperature whispering gallery resonators. The simulated cavity is virtually free from relaxation time and from flickering. The rest of the noise measurement system is based on a previous instrument [13] designed to operate in the VHF band, for only a short description is given here.

The circulator attenuates the carrier by some 25–30 dB, which is $|S_{31}|$ of the whole test circuit. Then, the carrier is attenuated as much as possible by vector addition of an opposite signal through the 10 dB coupler. A high carrier rejection is needed to prevent the amplifier from flickering by nonlinear up-conversion of the dc bias noise. The circulator noise sidebands

$$x(t) = n_1(t)\cos(2\pi\nu_0 t) - n_2(t)\sin(2\pi\nu_0 t)\ , \quad (1)$$

not affected by the interference mechanism, are amplified and down converted to baseband by the I-Q detector. Taking $\cos(2\pi\nu_0 t)$ as the phase reference, $n_1(t)$ and $n_2(t)$ are related to the amplitude and phase noise at the transmission port of the circulator by

$$\alpha(t) = \frac{n_1(t)}{V_0} \quad (2)$$

$$\varphi(t) = \frac{n_2(t)}{V_0}\ , \quad (3)$$

where $V_0 = \sqrt{2R_0P_0}$ is the peak voltage; the device loss, of some 0.2 dB, is neglected here. By inspection on Fig. 3, the detected signal is

$$\begin{bmatrix} v_1(t) \\ v_2(t) \end{bmatrix} = k_{\text{dsb}}\sqrt{P_0}\begin{bmatrix} \cos\psi & -\sin\psi \\ \sin\psi & \cos\psi \end{bmatrix}\begin{bmatrix} \alpha(t) \\ \varphi(t) \end{bmatrix}\ , \quad (4)$$

where k_{dsb} is the dual sideband (DSB) gain of the instrument. For the above equation to be true it is necessary that the two channels of the I-Q detector be equal and orthogonal. This is fixed by the matrix G, whose coefficients are determined with the Gram-Schmidt process [14]. Equation (4) also contains the arbitrary phase ψ that results from the circuit layout. The matrix B provides the frame rotation by which

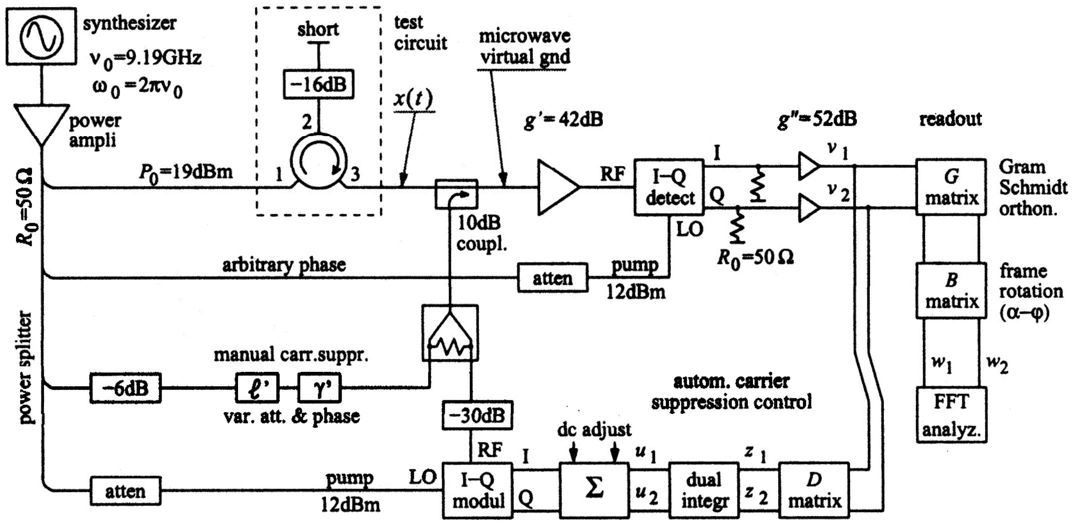

Figure 3: Scheme of the instrument.

the output signal $[w_1, w_2]^T$ (the superscript T stands for transposed) can be made proportional to $[\alpha, \varphi]^T$, or to the scalar projections of $[\alpha, \varphi]^T$ on any desired pair of Cartesian axes.

The carrier is automatically suppressed in closed loop by the I-Q modulator. The diagonalization matrix D transforms the control into two independent loops that work in Cartesian coordinates. Our previous paper [13] provides the theory and the details needed to put the control to work. Interestingly, the control is independent of the test circuit and of P_0.

The following information may be useful to duplicate our experiment. Owing to interference, the residual carrier does not exceed -80 dBm at the amplifier input. The amplifier is split into two stages with a 100 MHz bandwidth filter in between, which ensures linear—hence flicker-free—operation. The path ℓ'-γ' provides the fine carrier suppression needed to pull the control in. This is necessary because low flicker design imposes a low weight (-43 dB) of the signal from the I-Q modulator. The loop bandwidth is set to 0.3 Hz, so that the control has no effect on the measured spectra above a few Hz. The DSB gain is $k_{dsb} = 75$ dBV/$\sqrt{\text{mW}}$. This is measured by observing the output spectrum when the output of the test circuit (port 3) is replaced with a white noise source of known spectrum. Alternatively, a sinusoidal source of frequency ν_s a few kHz off ν_0 can be used, from which $k_{ssb} = k_{dsb}/\sqrt{2}$ is measured. Uncertainty is of some 1 dB, mainly due to the imperfect measurement of the reference signal. For best mechanical stability, the microwave section is mounted onto a standard 0.6×0.9 m^2 breadboard with M6 holes on a 25 mm pitch grid, of the type commonly used for optics.

3 Experimental Results

The background noise of the instrument is measured by replacing the test circuit with a 30 dB attenuator connected between ports 1 and 3. This simulates a nearly noiseless circulator that shows the same isolation of the real ones, while the rest of the instrument works in substantially unchanged conditions. The white noise is of -191 dB[rad^2]/Hz. This value is consistent with the expected value that results from $P_0 = 19$ dBm and from the noise figure $F = 2$ dB of the amplifier. As the detection frame is set by the matrix B, we use the notation dB[rad^2]/Hz to indicate that the unit of angle [rad] appears only in the measurement of φ. The residual flicker of the instrument is of some -180 dB[rad^2]/Hz at $f = 1$ Hz off the carrier.

All the spectra are measured in the 10–10^4 Hz span.

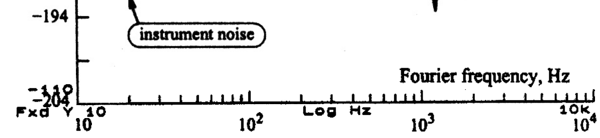

Figure 4: Example of measured spectrum.

factory and device type	ser. no.	$S_{\alpha\|\varphi}(1\,\mathrm{Hz})$ dB[rad^2]/Hz min.	max.	equivalent stability oscillator $\sigma_y(\tau)$, $\times 10^{-15}$	mechanical $\sigma_l(\tau)$, $\times 10^{-12}$ m
Aercomm J180-124	1320	−165.1	−162.6	22	36
Dorado 4CCC 10-1 ⋆	101	−171.6	−168.0	12	19
Trak C80124/A	E001	−165.9	−160.3	28	47
	E003	−165.7	−164.0	19	31
Narda CNA 8596	157	−170.3	−170.3	9	15
	158	−170.3	−169.1	10	17
Sivers Lab 7041X ‡	625	−176.0	−164.0	n.a.	n.a.
residual instrum. noise		< 180			4.9

Fig. 4 shows an example of spectrum. The $1/f$ coefficient of the circulator noise is extrapolated from the 10–100 decade, trusting in the usual polynomial approximation of the spectrum [15]. Being interested in $1/f$ noise, we could not measure the spectrum in the 1–10 Hz decade because a higher slope, f^{-2}, appears at $f \approx 1$ Hz. This is thought to be due to the fluctuations of the room temperature, which affect either the instrument or the circulator.

Six circulators have been measured, suitable to the 8–12 GHz range, and selected with the sole criterion that they are routinely used in our laboratory for ultrastable oscillators. Equal or similar circulators are used for the same purpose at the Jet Propulsion Laboratory, Pasadena, at the National Physical Laboratory, London, and at the University of Western Australia, Perth. Table 1 shows the results. Instead of calibrating the detection direction, we decided to search for the minimum and maximum flicker noise, i.e. the semi-axes of the noise ellipse, by sweeping the detection angle through the matrix B. For reference, we also measured an old waveguide isolator used in reverse mode.

The right but one column of Table 1 reports the two-sample (Allan) deviation $\sigma_y(\tau)$ of an oscillator that makes use of the circulator, under the conservative assumptions that: (i) the worst noise value is taken as it was true phase noise, regardless of ψ, and (ii) the merit factor is $Q = 2\times10^5$, which is typical of a 10 GHz whispering gallery resonator at room temperature. With $1/f$ frequency noise, σ_y is independent of τ. σ_y is calculated from

$$\sigma_y^2 = 2\ln(2)\frac{1}{4Q^2}S_\varphi^{\text{flicker}}(1\,\mathrm{Hz}) \; , \tag{5}$$

which combines the Leeson model [3] and the spectrum-to-variance transformation [15]. As a further consequence, the system of Fig. 3 can be used in the noise degeneration circuit of an oscillator. Assuming a stable cavity with $Q = 2\times10^5$, the circulator would limit the frequency stability to parts in 10^{-14}.

4 Mechanical Stability

The measurement if $1/f$ noise in circulators gives some information on the mechanical properties of microwave circuits. This unexpected benefit deserves a separate section.

A phase fluctuation $\varphi(t)$ can be regarded as an equivalent fluctuation of length $l(t) = \frac{\lambda}{2\pi}\varphi(t)$. Thus

$$l(t) = \frac{1}{2\pi}\frac{c'}{\nu_0}\varphi(t) \; , \tag{6}$$

where $c' \approx 0.8\,c$ is the wave velocity in the circuit. Having measured $S_\varphi(f)$, we know the spectrum of length fluctuation

$$S_l(f) = \left(\frac{1}{2\pi}\frac{c'}{\nu_0}\right)^2 S_\varphi(f) \; . \tag{7}$$

It is a common practice to represent a noise spectrum as the power-law

$$S(f) = \sum_\alpha h_\alpha f^\alpha \; , \tag{8}$$

where the term $h_1 f^{-1}$ is the flicker noise. $S(f)$ can be transformed into the Allan variance $\sigma(\tau)$ through

$$\sigma^2 = 2\ln(2)\,h_{-1} \; , \tag{9}$$

which holds for flicker noise. In the domain of time and frequency, (8) is usually written as $S_y(f) = \ldots$, and (9) as $\sigma_y^2(\tau) = \ldots$, where $y = \frac{1}{2\pi\nu_0}\frac{d\varphi(t)}{dt}$ is the fractional frequency fluctuation. Nevertheless, (9) is a mathematical property of spectra, which is independent of the physical quantity involved. Thus, we can calculate the Allan deviation

$$\sigma_l = \sqrt{2\ln(2)\,h_{-1}} \qquad (10)$$

of the equivalent length fluctuation l. Obviously, here h_{-1} is the $1/f$ coefficient of $S_\varphi(f)$, *not* of S_y.

Equation (10) explains the equivalent mechanical stability reported in the right column of Table 1). The measured values are parts in 10^{-11} m. One can figure out how small is this value by comparing it to the Bohr radius, which is $a_0 = 5.29\times10^{-11}$ m. Such a high stability would not be surprising in other domains, like that non-optical microscopy, where the attainable resolution is far below 1 pm. Yet, however careful our implementation is, it is based on standard microwave parts, which includes SMA connectors, PTFE-insulated semirigid cables, etc. The combined effect of magnetic fluctuations in the circulator, electric fluctuations (for example in the dielectric of cables), and mechanical fluctuations is in the 10^{-11} m range.

5 Discussion

The flicker noise of the circulator is due to the near-dc fluctuations that modulate the microwave carrier. There results amplitude noise $\alpha(t)$ and phase noise $\varphi(t)$ that exhibit power spectral densities proportional to $1/f$. The microwave spectrum is therefore proportional to $1/|\nu - \nu_0|$ around the carrier frequency ν_0. As a circulator shows a typical bandwidth of less than one octave, the dependence on carrier frequency is a minor concern.

Three possible noise mechanisms have been identified, namely:

- Magnetization noise. A change in the dc magnetic field affects the velocity of the slow wave and of the fast wave, for the "dark spot" of the interference changes its position with respect to the isolation port. Noise in the dc field results in microwave noise at the isolation port.

- Barkhausen effect. The ferrite consists of a set of small volumes, called Weiss domains, in which all the the magnetic momenta are aligned. As a consequence of thermal energy, boundary atoms flip between contiguous domains, which results in $1/f$ noise.

- Mechanical instability. A dimensional fluctuation results in random modulation of the microwave carrier, hence in noise. Besides the electrical length fluctuation, mechanical instability can affect the dc bias through the reluctance of the magnetic circuit.

A major difficulty in ascribing the observed results to the above phenomena, or to other ones, derives from the lack of information about the internal structure of circulators.

References [11, 12] report $S_\alpha(1\,\mathrm{Hz}) = -149\,\mathrm{dB/Hz}$ and $S_\varphi(1\,\mathrm{Hz}) = -154\,\mathrm{dBrad^2/Hz}$, which is 10–20 dB higher than our values. The circulator, similar to our ones, was measured at the same frequency and driving power as we did, but in forward mode. The difference in the measured noise, 10–20 dB, makes one think that there is a systematic difference between transmission and isolation noise, and that the fluctuations are lower at the isolation port. Magnetic fluctuations seem to be correlated over the ferrite bulk, and to cancel out at the isolation port, where destructive interference takes place. This conjecture could be checked by measuring the noise of each circulator in forward mode. Unfortunately, at the present time this can not be done because the carrier suppression requires an additional attenuator and a phase shifter that work at $P_0 = 19$ dBm. Due to the power, the noise of these devices limits the low-frequency sensitivity. It should be remarked that the results reported in [11, 12] could have been obtained only by cascading a few devices.

6 Conclusions

The flicker noise of some circulators has been measured in isolation mode, which is relevant to ultra-stable oscillators. The experiments have been made possible by the low residual flicker of the described instrument. The results give some preliminary indications about the noise mechanism of the circulator and on the mechanical properties of microwave assemblies.

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References

[1] R. V. Pound, "Electronic frequency stabilization of microwave oscillators," *Rev. Sci. Instr.*, vol. 17, pp. 490–505, Nov. 1946.

[2] Z. Galani, M. J. Bianchini, R. C. Waterman, Jr., R. Dibiase, R. W. Laton, and J. Bradford Cole, "Analysis and design of a single-resonator GaAs FET oscillator with noise degeneration," *IEEE Trans. Microw. Theory Tech.*, vol. 32, pp. 1556–1565, Dec. 1984.

[3] D. B. Leeson, "A simple model of feed back oscillator noise spectrum," *Proc. IEEE*, vol. 54, pp. 329–330, 1966.

[4] H. Bosma, "On stripline Y-circulation at UHF," *IEEE Trans. Microw. Theory Tech.*, vol. 12, pp. 61–72, Jan. 1964.

[5] C. E. Fay and R. L. Comstock, "Operation of the ferrite junction circulator," *IEEE Trans. Microw. Theory Tech.*, vol. 13, pp. 15–27, Jan. 1965.

[6] J. Helszajn, "Complex gyrator circuits of planar circulators using higher order modes in a disk resonator," *IEEE Trans. Microw. Theory Tech.*, vol. 31, pp. 931–938, Nov. 1983.

[7] J. Helszajn, "The higher mode stripline circulator using a gyromagnetic disk resonator," *Microw. Engin.*, pp. 35–45, May 2002.

[8] A. E. Siegman, "Thermal noise in microwave systems. Part 1: Fundamentals," *Microw. J.*, vol. 4, pp. 81–90, Mar. 1961.

[9] A. D. Sutherland, "On the inherent noise of an ideal two-port isolator," *IEEE Trans. Microw. Theory Tech.*, vol. 30, May 1982.

[10] D. P. Tsarapkin, "Low phase noise sapphire disk dielectric resonator oscillator with combined stabilization," in *Proc. Freq. Contr. Symp.*, vol. 48, pp. 451–458, 1994.

[11] E. N. Ivanov, M. E. Tobar, and R. A. Woode, "A study of noise phenmena in microwave components using an advanced noise measurement system," *IEEE Trans. Ultras. Ferroel. and Freq. Contr.*, vol. 44, pp. 161–163, Jan. 1997.

[12] R. A. Woode, E. Ivanov, and M. E. Tobar, "Application of the interferometric noise measurement technique for the study of intrinsic fluctuations in microwave isolators," *Meas. Sci. Technol.*, vol. 9, pp. 1593–1599, Sept. 1998.

[13] E. Rubiola and V. Giordano, "Advanced interferometric phase and amplitude noise measurements," *Rev. Sci. Instr.*, vol. 73, pp. 2445–2457, June 2002.

[14] R. A. Horn and C. R. Johnson, *Matrix Analysis.* Cambridge University Press, 1985.

[15] J. Rutman, "Characterization of phase and frequency instabilities in precision frequency sources: Fifteen years of progress," *Proc. IEEE*, vol. 66, pp. 1048–1075, Sept. 1978.